

INVESTIGATION OF FIT OF BETA AND NORMAL  
DISTRIBUTIONS TO A PRODUCT OF BETA  
DISTRIBUTION

by

John Stager Foard



# United States Naval Postgraduate School



## THESIS

INVESTIGATION OF FIT OF BETA AND NORMAL  
DISTRIBUTIONS TO A PRODUCT OF BETA DISTRIBUTION

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Investigation of Fit of Beta and Normal  
Distributions to a Product of Beta Distributions

by

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## ABSTRACT

Excessive costs prohibit the Department of Defense and industry from testing entire systems to obtain estimates of system reliability. Thus, individual component tests must be combined in such a way as to yield an accurate estimate of the system reliability.

Assuming each component reliability to have a prior distribution which is beta distributed, Bayesian techniques result in the posterior distributions also being beta distributed. The system reliability for a series system would then be the product of the posterior distributions.

Computer simulation techniques were used to determine the system reliability distribution for a series system of beta distributed components. Method of Moments techniques were then used to fit beta and normal distributions to the system distribution. The twentieth percentile points of the fitted and the actual distributions were then compared as a measure of accuracy of the fit.

The fit of the beta distribution proved to be accurate for all parameter ranges and number of components in each system. The fit of the normal distribution was accurate only when used with limited parameter values.





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## I. INTRODUCTION AND SUMMARY

### A. INTRODUCTION

The purpose of this study was to analyze the accuracy of fitting a product of beta distributions with beta and with normal distributions using computer simulation techniques. If successful, a normal or beta distribution could be used to fit the posterior distributions in Bayesian techniques to the reliability of a series system where the component prior distributions are also beta distributed.

### B. MEASURE OF ACCURACY

The Bayesian  $100(1-\alpha)\%$  lower confidence limit for the system reliability,  $R_s$ , is the  $100\ \alpha^{\text{th}}$  percentile point of the distribution of  $R_s$ . Therefore, the difference between the  $100\ \alpha^{\text{th}}$  percentile points of the actual and the fitted distributions was selected as a measure of the accuracy of the fit.

### C. SUMMARY OF RESULTS

When the method of moments technique was used to fit a normal distribution to the distribution of the reliability of a series system, the fit was accurate for a limited number of cases. The data depicted in Table III indicates that the normal distribution fit can offer a simple and accurate method of estimating the system reliability.

When the constraints of the fitted normal cannot be met or if an easy method of obtaining a percentile point



of a beta distribution is available, the beta fit should be used.





## II. THE PROBLEM

### A. BACKGROUND

Due to the costliness of mission testing entire systems, the Department of Defense must rely on individual component tests to estimate system reliability. Among other things, the accuracy of this estimate is dependent upon the fit of a specific distribution to actual distributions. It is often necessary due to mathematical and/or statistical complications to assume a distribution which only approximates the actual distribution. A secondary yet important consideration is the manner in which the component test data is utilized in determining the estimated system reliability.

Bayesian techniques offer an acceptable solution to the test data utilization. In Bayesian analysis, the beta distribution with parameters  $a$  and  $b$  is usually chosen as the prior distribution for component reliability. The parameters  $a$  and  $b$  relate to attribute type data. A problem arises when trying to combine the component beta distributions to get a confidence interval on system reliability. In particular, the problem exists on how to construct methods for estimating the system reliability when each of the component reliabilities is assumed to be beta distributed.

### B. BETA DISTRIBUTION AND BAYESIAN TECHNIQUES

#### 1. Beta Distributions

The beta distribution with parameters  $a$  and  $b$  has the following probability function:



$$f(x) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} x^{a-1}(1-x)^{b-1} \quad a, b, x > 0.$$

The mean,  $\mu$ , and variance,  $\sigma^2$ , are computed by the following formulae:

$$\mu = \frac{a}{a+b}$$

$$\sigma^2 = \frac{ab}{(a+b)(a+b+1)}$$

## 2. Bayesian Techniques

The usefulness of beta prior distributions and Bayesian techniques can be depicted by the following example. Suppose  $n$  components of one type are mission tested and further suppose each component is beta distributed,  $\beta(x; a, b)$ . Let  $s$  be the number of components which do not fail. The  $100(1-\alpha)\%$  lower Bayesian confidence limit on component reliability is the solution for  $P$  in the equation

$$\alpha = \int_0^P \beta(x; a+s, a+b+n-s+1) dx.$$

That is, the posterior distribution of component reliability is  $\beta(x; a+s, b+n-s)$ . Thus, by the use of beta distributions and Bayesian techniques, both past and present test data can readily utilized in reliability calculations.



### III. SOLUTION

#### A. ACTUAL SYSTEM DISTRIBUTION

The actual system distribution to which the beta and normal distributions were fitted was generated on the IBM 360 Computer at the United States Naval Postgraduate School, Monterey, California. The main program was used with slight modifications to evaluate the  $100\alpha^{\text{th}}$  percentile points of the fitted beta distributions. The required subroutines were obtained from the Computer Center Library.

##### 1. Main Program

The main program was constructed to randomly generate a reliability value for each component of the series systems. Assuming the component distributions to be independent, the system reliability,  $R_s$ , would then be the product of the component reliabilities,  $R_i$ ; i.e.,  $R_s = \prod_{i=1}^K R_i$ . Thus, by multiplying the generated component reliability values, a value of the system reliability was obtained. This process was repeated to obtain 500 values of  $R_s$ . The values were ordered to obtain a simulated distribution of  $R_s$ . The one hundredeth number of this distribution was then assumed to be the twentieth percentile point of the actual system reliability distribution.

##### 2. Subroutines

The Subroutine BDTR was used to obtain values of the cumulative distribution function of the beta distributions. The subroutine is contained in R. E. Bargmann and S. P. Ghosh,



Statistical Distribution Programs for a Computer Language, I.B.M. Research Report RC-1094. It has a maximum error of  $10^{-5}$  for values of a and b between 0.5 and  $10^5$ . Subroutine BDTR used subroutine NDTR, CDTR, and DLGAM in its computations.

## B. METHOD OF MOMENTS BETA FIT

The supposition is that the system reliability,  $R_s$ , is beta distributed with parameters A and B where  $R_s = \prod_{i=1}^K R_i$  and the  $R_i$ 's are beta distributed with parameters  $a_i$  and  $b_i$  respectively. Using the method of moments technique, the mean,  $\mu$ , and variance,  $\sigma^2$ , of the system distribution are as follows:

$$\begin{aligned}\mu &= E(R_s) = \prod_{i=1}^K E(R_i) \\ &= \frac{A}{A+B} = \prod_{i=1}^K \frac{a_i(a_i+1)}{(a_i+b_i)(a_i+b_i+1)}.\end{aligned}$$

$$\sigma^2 = \frac{AB}{(A+B)^2(A+B+1)} = E(R_s)^2 - [E(R_s)]^2.$$

In addition,

$$E(R_s^2) = \frac{A(A+1)}{(A+B)(A+B+1)} = \prod_{i=1}^K \frac{a_i(a_i+1)}{(a_i+b_i)(a_i+b_i+1)}.$$

Letting

$$\mu_1 = \prod_{i=1}^K \frac{a_i}{a_i+b_i}$$

and

$$\mu_2 = \prod_{i=1}^K \frac{a_i(a_i+1)}{(a_i+b_i)(a_i+b_i+1)},$$





A and B can be determined:

$$A = \frac{(\mu_1 - \mu_2) \mu_1}{\mu_2 - \mu_1^2}$$

$$B = \frac{(1 - \mu_1)(\mu_1 - \mu_2)}{\mu_2 - \mu_1^2}$$

Subroutine BDTR was then used to determine the twentieth percentile point of the beta distribution with parameters A and B as calculated.

### C. METHOD OF MOMENTS NORMAL FIT

Define  $Q_i = 1 - R_i$  and assume  $R_i$  is large."  $Q_i$  is a beta distributed random variable with parameters  $b_i$  and  $a_i$ .  $R_s$  is equal to  $\sum_{i=1}^K (1 - Q_i)$ . Since  $Q_i$  is "small," expanding  $\sum_{i=1}^K (1 - Q_i)$  and neglecting the cross products,  $R_s = 1 - \sum_{i=1}^K Q_i$  and  $1 - R_s = \sum_{i=1}^K Q_i$ . The supposition is now that  $1 - R_s$  is normally distributed, with parameter  $\mu$  and  $\sigma^2$  since it is a sum of random variables. Using method of moments techniques,

$$\mu = E(1 - R_s) = E\left(\sum_{i=1}^K Q_i\right) = \sum_{i=1}^K E(Q_i) = \sum_{i=1}^K \frac{b_i}{a_i + b_i}$$

and

$$\begin{aligned} \sigma^2 &= \text{var}(1 - R_s) = \text{var}\left(\sum_{i=1}^K Q_i\right) = \sum_{i=1}^K \text{var}(Q_i) \\ &= \sum_{i=1}^K \frac{a_i b_i}{(a_i + b_i)^2 (a_i + b_i + 1)}. \end{aligned}$$

The twentieth percentile point of  $R_s$  is the same as the eightieth percentile point of  $1 - R_s$ . The eightieth percentile point of  $1 - R_s$  is  $\sigma Z_{.8} + \mu$  where  $Z_{.8}$  is the eightieth percentile point of the Standard Normal Distribution.



## IV. RESULTS

### A. ACTUAL SYSTEM DISTRIBUTION

The actual system distributions were generated for series systems of fifteen and twenty-five components using the nineteen sets of parameters as listed in the first three tables. It was assumed that all components in the systems would have the same parameter values. The twentieth percentile points of these distributions are listed in Tables I and II.

Due to the number of runs required and to the length of computer time required to call subroutine BDTR, the number of samples forming these distributions was limited to five hundred. This limitation resulted in the possible variance of the twentieth percentile point of a given distribution by as much as .003 when a different series of random numbers were used to generate the same distribution. Variations of this amount would make a significant change in column five of Tables I and II. For example, using  $a=1000.0$  and  $b=5.0$  in Table I, the difference between the actual twentieth percentile point and the fitted beta point was only .002. Yet, this difference resulted in an error of 4.6 percentage points ( $20.0-15.4=4.6$ ).

### B. METHOD OF MOMENTS BETA FIT

Using the formulae developed previously to obtain the system parameters A and B, the twentieth percentile point of the system distribution was calculated by computer using



subroutine BDTR. Considering the maximum error of BDTR, the data of column four in Tables I and II should be more accurate than that of the actual distribution.

Examination of columns three and four of Tables I and II shows that the beta fitted distribution provides a more accurate estimate of the actual distribution for the system of twenty-five components than for the fifteen. However, even for the smaller system, the method does result in reasonably accurate estimates of the twentieth percentile point particularly if a variance of .003 is introduced into the actual distribution percentile point. It is interesting to note that the method appears to be about the same degree accurate throughout the parameter ranges for both systems. Due to the steepness of the curves involved, the small percentage error figures can be misleading. The actual percentile points (column five) might offer a better measure of accuracy. These points indicate that in most of the cases the error is conservative and that the approximation actually represents the lower limit of a higher degree of confidence than eighty percent.

#### C. METHOD OF MOMENTS NORMAL FIT

The results of the normal fit are depicted in Table III. As can readily be seen in this table, the normal fit is not accurate when the ratio of  $a$  to  $b$  is less than 500 to 1. Since these parameters can be related to the number of failures and number of successes, these results tend to prove the initial assumption made when deriving this method.



This assumption was that  $R_i$  was "large." The results indicate that "large" means one or less failures out of 500 mission tested components. Given these parameter constraints, the normal fit is quite accurate. Due to the limited range of parameters, this method was not investigated further.

#### D. BETA FIT WITH VARIOUS PARAMETERS

The process of fitting the beta distribution to the system reliability was successful enough to investigate the accuracy of the method when the components have different parameter values. Table IV depicts these results. The comparison was made for systems of fifteen and twenty-five components and for two different general parameter ranges. The limitations of computer core suggested that no more than five different distributions be used. Thus, in the fifteen component system, there are three components with each set of parameters; and in the twenty-five component system, there are five components with each set of parameters.

The results indicate that the method of moment fitted beta distribution is equally accurate when there are mixed distributions in the systems.





TABLE I

This table lists the results of the method of moments beta fit and the actual distribution for a series system of fifteen components. All distributions in the system have the same parameters. Columns one and two list the a and b parameters respectively. Column three is the twentieth percentile point of the actual distribution. Column four is the twentieth percentile point of the beta fitted distribution. Column five lists the percentile point of the actual distribution that the number in column four represents. Column six is the percent error which is the difference between column three and column four divided by column three.

<u>a</u>	<u>b</u>	<u>Actual</u>	<u>Beta Fit</u>	<u>Act. %</u>	<u>Error %</u>
10.0	0.5	0.372	0.372	20.0	0.0
10.0	1.0	0.168	0.163	18.8	3.0
30.0	0.5	0.768	0.723	4.4	5.9
30.0	1.0	0.545	0.547	20.8	0.3
30.0	2.0	0.328	0.323	17.4	1.6
50.0	0.5	0.825	0.824	19.4	0.1
50.0	1.0	0.700	0.696	18.8	0.6
50.0	2.0	0.529	0.505	7.8	4.5
100.0	0.5	0.913	0.908	16.4	0.6
100.0	1.0	0.839	0.834	17.6	0.6
100.0	2.0	0.716	0.709	16.4	1.0
500.0	0.5	0.981	0.981	20.1	0.0
500.0	1.0	0.965	0.964	17.8	0.0
500.0	2.0	0.936	0.933	13.0	0.5
500.0	5.0	0.850	0.849	17.4	0.2
1000.0	0.5	0.990	0.990	20.1	0.0
1000.0	1.0	0.983	0.982	17.6	0.0
1000.0	2.0	0.967	0.966	17.6	0.0
1000.0	5.0	0.923	0.921	15.4	0.2



TABLE II

This table lists the results of the method of moments beta fit and the actual distribution for a series system of twenty-five components. All distributions in the system have the same parameters. Columns one and two list the a and b parameters respectively. Column three is the twentieth percentile point of the actual distribution. Column four is the twentieth percentile point of the beta fitted distribution. Column five lists the percentile point of the actual distribution that the number in column four represents. Column six is the percent error which is the difference between columns three and four divided by column three.

<u>a</u>	<u>b</u>	<u>Actual</u>	<u>Beta Fit</u>	<u>Act. %</u>	<u>Error %</u>
10.0	0.5	0.205	0.207	20.8	0.8
10.0	1.0	0.055	0.054	19.4	1.8
30.0	0.5	0.650	0.597	2.6	8.2
30.0	1.0	0.379	0.379	20.0	0.0
30.0	2.0	0.160	0.160	20.1	0.0
50.0	0.5	0.733	0.735	20.8	0.3
50.0	1.0	0.560	0.559	19.6	0.2
50.0	2.0	0.358	0.331	5.0	7.5
100.0	0.5	0.860	0.857	17.6	0.3
100.0	1.0	0.747	0.747	20.0	0.0
100.0	2.0	0.575	0.574	19.4	0.2
500.0	0.5	0.970	0.970	20.8	0.0
500.0	1.0	0.944	0.944	19.6	0.0
500.0	2.0	0.897	0.894	14.2	0.3
500.0	5.0	0.765	0.765	21.0	0.0
1000.0	0.5	0.985	0.985	21.0	0.0
1000.0	1.0	0.971	0.971	19.0	0.0
1000.0	2.0	0.945	0.946	20.8	0.1
1000.0	5.0	0.875	0.875	18.0	0.0



TABLE III

This table lists the results of the method of moments normal fit and the actual distribution for a series system of twenty-five components. All distributions in the system have the same parameters. Columns one and two list the a and b parameters respectively. Column three is the twentieth percentile point of the actual distribution. Column four is the twentieth percentile point using the techniques described previously.

<u>a</u>	<u>b</u>	<u>Actual</u>	<u>Normal Fit</u>
10.0	0.5	0.205	- 0.454
10.0	1.0	0.055	- 1.62
30.0	0.5	0.650	0.49
30.0	1.0	0.379	0.062
30.0	2.0	0.160	- 0.739
50.0	0.5	0.733	0.694
50.0	1.0	0.560	0.429
50.0	2.0	0.358	- 0.072
100.0	0.5	0.860	0.846
100.0	1.0	0.747	0.711
100.0	2.0	0.575	0.452
500.0	0.5	0.970	0.969
500.0	1.0	0.944	0.942
500.0	2.0	0.897	0.888
500.0	5.0	0.765	0.734
1000.0	0.5	0.985	0.984
1000.0	1.0	0.971	0.971
1000.0	2.0	0.945	0.944
1000.0	5.0	0.875	0.866



TABLE IV

This table lists the results for method of moments beta fit using different parameters in the component distributions. Columns one and two are a and b parameters used. Column three is the number of components in the system. Five different sets of parameters are used. In the fifteen component system, there are three components with the same set of parameters. In the twenty-five component system, there are five components with the same set of parameters. Column four is the twentieth percentile point of the actual distribution. Column five is the twentieth percentile point of the fitted beta distribution.

<u>a</u>	<u>b</u>	<u>k</u>	<u>Actual</u>	<u>Beta Fit</u>
50.0	0.5			
100.0	0.5	15	0.822	0.820
100.0	1.0			
100.0	2.0	25	0.739	0.728
500.0	5.0			
500.0	1.0			
500.0	2.0	15	0.966	0.966
1000.0	0.5			
1000.0	1.0	25	0.946	0.947
1000.0	2.0			





TABLE V

Using the method of moments beta fit,  $R_s$  was assumed to be beta distributed with parameters A and B. This table lists the parameters A and B for the various numbers of system components and component parameters a and b. Columns one and two are the a and b parameters respectively. Columns three and six are the number of components in the system. Columns four and five are the A and B values for the variables of columns one, two, and three. Columns seven and eight are the A and B values for the variables of columns one, two, and six.

<u>1.a</u>	<u>2.b</u>	<u>3.k</u>	<u>4.A</u>	<u>5.B</u>	<u>6.k</u>	<u>7.A</u>	<u>8.B</u>
10.0	0.5	15	7.2	7.5	25	5.9	14.0
10.0	1.0	15	5.5	17.5	25	3.8	37.8
30.0	0.5	15	26.8	7.5	25	24.8	12.7
30.0	1.0	15	24.1	15.3	25	20.8	26.4
30.0	2.0	15	19.8	32.3	25	15.3	61.4
50.0	0.5	15	46.7	7.5	25	44.5	12.6
50.0	1.0	15	43.7	15.1	25	39.8	25.5
50.0	2.0	15	38.5	30.9	25	32.4	54.0
100.0	0.5	15	96.6	7.5	25	94.3	12.5
100.0	1.0	15	93.3	15.0	25	88.9	25.1
100.0	2.0	15	87.4	30.2	25	79.6	51.0
500.0	0.5	15	500.7	7.6	25	498.9	12.6
500.0	1.0	15	490.7	14.9	25	485.4	24.9
500.0	2.0	15	485.9	30.0	25	476.1	50.0
500.0	5.0	15	465.3	74.9	25	443.4	125.2
1000.0	0.5	15	1029.9	7.8	25	1046.6	13.2
1000.0	1.0	15	976.0	14.7	25	969.2	24.5
1000.0	2.0	15	993.8	30.2	25	981.4	50.3
1000.0	5.0	15	966.8	75.1	25	942.9	125.2
50.0	0.5	3			5		
100.0	0.5	3			5		
100.0	1.0	3	89.98	16.03	5	85.27	26.79
100.0	2.0	3			5		
500.0	5.0	3			5		
500.0	1.0	3			5		
500.0	2.0	3			5		
1000.0	0.5	3	578.16	16.69	5	576.65	28.02
1000.0	1.0	3			5		
1000.0	2.0	3			5		



# MAIN PROGRAM FOR FIVE DIFFERENT COMPONENT TYPES

```

DIMENSION RSHAT(500),XARAY(5,750),PARAY(5,750),NUM(5)
Z=URN(0)
DATA NITER,I37,I3,KCOMP,PROD,NREP,ALPER,I31/
DO 555 I32=1,5
400 READ(5,401)ALFA,BETA,NSTAR,NSTOP,N,CFACT
401 FORMAT(2F6.1,2I7,I1,F8.7)
I2=1
DO 402 I21=NSTAR,NSTOP,N
X=I21*CFACT
CALL BDTR(X,ALFA,BETA,P,DOUT,IER)
XARAY(I32,I2)=X
PARAY(I32,I2)=P
I2=I2+1
402 CONTINUE
NUM(I32)=I2-1
555 CONTINUE
DO 222 I1=1,NITER
I32=1
DO 333 I4=1,KCOMP
FLY=URN(1)
49 IF(FYL-PARAY(I32,I3))50,50,51
51 I3=I3+1
IF(I3.EQ.NUM(I32))GO TO 60
GO TO 49
50 A=PARAY(I32,I3)-FYL
IF(I3.EQ.1)GO TO 60
IT3=I3-1
B=FYL-PARAY(I32,IT3)
IF(A-B)60,60,61
61 RIHAT=XARAY(I32,IT3)
GO TO 62
60 RIHAT=XARAY(I32,I3)
62 PROD=PROD*RIHAT
I3=1
AI4=I4
AM=AI4/I37
MM=AM+.2
M=I4/I37
IF(MM.EQ.M)I32=I32+1
333 CONTINUE
RSHAT(I1)=PROD
PROD=1.
222 CONTINUE
J=0
8 IF(J.EQ.NITER-1)GO TO 11
I=1
J=0
9 IF (RSHAT(I).GT.RSHAT(I+1))GO TO 10
J=J+1
I=I+1
IF(I.EQ.NITER)GO TO 8
GO TO 9
10 RTEMP=RSHAT(I)
RSHAT(I)=RSHAT(I+1)
RSHAT(I+1)=RTEMP
I=I+1
IF(I.EQ.NITER)GO TO 8
GO TO 9
11 CONTINUE
STOP
END

```



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13. ABSTRACT <p>Excessive costs prohibit the Department of Defense and industry from testing entire systems to obtain estimates of system reliability. Thus individual component tests must be combined in such a way as to yield an accurate estimate of the system reliability.</p> <p>Assuming each component reliability to have a prior distribution which is beta distributed, Bayesian techniques result in the posterior distributions also being distributed. The system reliability for a series system would then be the product of the posterior distributions.</p> <p>Computer simulation techniques were used to determine the system reliability distribution for a series system of beta distributed components. Method of Moments techniques were then used to fit beta and normal distributions to the system distribution. The twentieth percentile points of the fitted and the actual distributions were then compared as a measure of accuracy of the fit.</p> <p>The fit of the beta distribution proved to be accurate. for all parameter ranges and number of components in each system. The fit of the normal distribution was accurate only when used with limited parameter values.</p>			



## KEY WORDS

## LINK A

## LINK B

## LINK C

ROLE

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Beta Distributions

Bayesian Techniques

Method of Moments Techniques







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